JAKOB MELØE

Dialogue on the Hypothetical Character of Logical Analysis

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It is not unrealistic (nor would it matter much if it were) to imagine the two sentences “Norway is a democracy” and “Norway is not a democracy” to be printed within the same text (with some space between them).

If the phenomenon is noticed by a logical analyst his reaction may well be such as to make this an adequate rational reconstruction of it: the two sentences are pictured on the same line with an “and” between them; the conjunction is symbolized as “p · ~p”; the analyst remembers that “p · ~p” is the simplest contradiction in the sentential calculus (in the usual formulation of it); and the analyst asserts: there is at least one contradiction in the text. (Something of the sort may actually occur if the analyst is asked for a *justification* of his verdict.)

But even this innocent little piece of logical analysis is impure with empirical presuppositions. Notice, e. g., that our analyst asserted: there is at least one contradiction *in the text*. But the formula “p · ~p” is not printed in the text, and it was *this* formula that the analyst remembered to be a contradiction.

If a logical analyst is asked to justify his verdict “S is a contradiction” or “S is analytic” (where S is a natural language sentence), one frequent type of answer is “Because S can be turned into a formal contradiction”, like “p · ~p”, “Because S can be turned into a tautology”, like “p ∨ ~p”, “p ⊃ p”, etc. Such an answer *indicates* a justification. However, justification by reference to formal systems is no safer than the *prima facie* more obscure references to the meanings of the terms and sentences involved, since, it will be argued, the former is relevant only if the latter is included.

We arrange a dialogue between *LOG* and *SEM* on the topic. *LOG* is a philosopher or scientist who employs formal systems in his examination of arguments and doctrines which are formulated in natural languages, i. e. he is a *logical analyst* in one frequent sense of that term. *SEM* is, e.g., the present author, who, in the year 1957, feels little resistance against being classified as a member of the Oslo group of empirical semantics. (The points made in sections 3, 4 and 6 below are typical of the group.)

1. *LOG*: There is a least one contradiction of the form “p · ~p” in the text.

SEM: What do you mean by “of the form “p · ~p””? The symbol “p · ~p” does not itself occur in the text, and there is no substitution instance of it either.

LOG: I know. But let the first “p” stand for a proposition, and the second “p” for the same proposition. Also, let “p” with the figure “~” in front of it (“~p”) stand for the negation of p: i. e. the proposition that *must* be false if p is true, and *must* be true if p is false. Finally, let the dot (“·”) stand for conjunction. A conjunction is a combination of two or more propositions such that if each constituent proposition is true, then the combination of them must be true, and if one or more constituents are false, then the combination must be false.

Then, given this *interpretation* of the formula "p · ~p", there is at least one instance of the formula thus interpreted in the text. And since the formula is a contradiction, there is at least one contradiction in the text.

SEM: Excellent. But is there in the text a conjunction of one proposition and its negation?

LOG: There is the sentence "Norway is a democracy and Norway is not a democracy".

SEM: Is there?

LOG: To be more accurate, there are the two sentences "Norway is a democracy" and "Norway is not a democracy". I skipped one step in the deduction.

SEM: The deduction?

LOG: If *p* is asserted and *q* is asserted, we can infer the assertion of $p \cdot q$. It is asserted that Norway is a democracy. It is also asserted that Norway is not a democracy. This entails the assertion that Norway is a democracy and not a democracy. And this we write: "Norway is a democracy and Norway is not a democracy".

SEM: I know that if at some place earlier than *j* in a deduction there is a well-formed formula P, and at some other place earlier than *j* there is a well-formed formula Q, then at place *j* one may write P · Q. Such are the rules of the game. But who told you that the author of our text is playing the same game?

LOG: He must.

SEM: I agree that he must in order for you to be able to make that deduction. It is perhaps even advisable to stick to conventions, which make it possible for us to make such deductions. But that our author *does* stick to such conventions is only a probable hypothesis.

LOG: I guess you are free to talk about 'only probable hypotheses'. What you actually do is to prescribe a given role (out of several possible ones) for the sentential calculus. [SEM (to himself): what you actually do when you say "what you actually do is to prescribe a given role (out of several possible ones) for the sentential calculus" is to prescribe a given role (out of several possible ones) to my words]. When propositions take the place of the "p"s and "q"s, you turn the rules and axioms of the calculus into hypotheses about the presence of such rules and axioms in the natural language, or some specified part of it (the part being, e. g., certain types of situations wherein the language is used, e. g., scientific communication).

One usually gives the calculus a very different role. So when I say that I deduce an asserted conjunction *p and q* from the assertion of *p* and the assertion of *q*, I do not make any hypotheses about the rules or conventions which our author may happen to follow (on whatever basis we decide what those rules are (in whatever sense of "rule")). I make a simple inference of logic. A rule of inference like, e. g., P, Q ⊥ P · Q is not restricted to the sentential calculus as a formal system, and applicable to a particular text only if it is empirically true that the rules of inference for the calculus are isomorphic to the rules of inference used in that particular text. Hence, I don't hypothesize that the logic of the text is isomorphic to the propositional calculus — rather, I announce that it shall be so.

Since this legislation is in fact accepted by quite a few people, there are quite a few people who accept the deduction of *p and q* from *p, q*. If a given author follows different conventions, well, then in a sense it is not *his* text we read when we read what he has written. (And we are under no *obligation* to read *his* text either.) The text may be more consistent in his logic *i* than in our logic *j*, or it may be less consistent — if you don't postulate that no text is inconsistent.

SEM: I don't suppose you mean to imply that the isomorphy between, e.g., Boolean algebra and electrical circuits is normative. Yet, it is, perhaps, better to take the isomorphy between, e.g., the sentential calculus as a formal system and the logic of (a particular application of) natural language as normative. Hence I withdraw the first candidate in my list of empirical presuppositions in your little piece of logical analysis.

2. Before I present the second candidate, let me make a comment on your very last remark. — I don't postulate that a text is never inconsistent. But I do postulate that a philosopher or scientist never intends to assert a contradiction or a tautology. Take the latter. (And by "tautology" I mean the same as "sentence which, whatever its form, expresses an analytic proposition".) There is, perhaps, one way in which a tautology could be said to inform, namely by exhibiting, not speaking about, some definitional property of its logical subject (if the proposition is of the subject-predicate form). In order to do so, however, it must be recognized as a tautology offhand (if not, it can be judged to be one only by presupposing the information it was intended to yield). But how can one know offhand whether a given sentence is intended as a tautology or not? It may have some visible mark signalizing its status. But then it is synonymous to, or equivalent in function to, a sentence of the type "*In our terminology the sentence "…" is a tautology*", which is not a tautology.

Tautologies proper give us no information whatsoever, and quasi-tautologies (i. e. marked tautologies) give us only indeterminate information about definitional properties. Whereas a given definition determines uniquely a set of tautologies, a given tautology is determinable by more than one definition. (If I know that the term "east" within a given text is synonymous to "where the sun rises", then I know that, within that text, each specimen of the sentence "The sun rises in the east" is a tautology. But if I know that the sentence "The sun rises in the east" is a tautology, do I then know what it is which has at least one definitional property exhibited? (Is it the sun, or the rising of the sun, or the east, or what?))

The postulate (or rule of interpretation), then, is: To the extent that a given author intends to inform, he does not intend to assert tautologies proper (since tautologies proper do not inform). (So there is a sense in which a mathematical formula does not inform — and there is a sense in which it does, e.g., if it is interpreted as a statement about itself; just as a given position in chess can be interpreted as a statement about itself, that one can reach it from the opening position by legitimate moves.)

However, a given text may contain un-intentional (or un-noticed) tautologies or contradictions. Neither of the two players in a game of chess intends to play his king into a position of checkmate, and yet, checkmate is the usual ending of a game.

So I take into account the possibility of contradictions. Only we ought to notice that when we identify something as a contradiction, we presuppose a number of empirical premises to be true.

3. Returning to your particular analysis: If our author asserts p, and also asserts q, then we agree to make him responsible for the assertion of p and q. But (and this is my second candidate) how do we know that he asserts p, and that he asserts q? You take it for granted that he asserts both of them. But perhaps he does not. Perhaps he asserts only one of them.

LOG: There is of course the chance that I overlooked some quotes, i. e., that one of the sentences is *mentioned*, not asserted.

SEM: I think I ought to reformulate the point I am trying to make. One classification of empirical sentences (or propositions) is into *observation-sentences* and *hypothetical sentences* (universal or particular (synthetic) sentences that refer to non-observables). The distinction, as I know it, is rather vague, i. e. on a number of relevant points the decision mechanism of the definiens (ignoring at the moment the multiplicity of definientia) does not react — and either one is not able to sort *each* sentence, or one sorts on the basis of *ad hoc* decisions. But the vagueness of the distinction does not affect its job here. It is quite clear that you do not *see* the meaning of a given word. I mean, the meaning is not to be found printed on your retina as is (in some sense) the physical shape of the word. A sentence about the physical shape of a word is therefore classified as an observation-sentence, whereas a sentence about the meaning of a given word is classified as a hypothetical sentence. (This is a rough way of dealing with delicate matters, but the topic of *seeing* is only peripheral to our discussion.)

Well, then my point is: *In order to establish the result of your logical analysis as a conclusion in a deduction from premises, you have to include in your set of premises sentences which are not only empirical, but also non-observational, i. e. hypothetical.*

From your experience with chess and logistic symbol games you know that it is possible to overlook an illegitimate move. [As to logistic games, a statement that a particular sequence of well-formed formulas is, in fact, a proof may be, construed as a conjunction of observation statements — if the rules of inference are interpreted and applied, but not spoken about. (The restriction is not arbitrary. For one thing — and apart from more serious consequences — if a statement about the meaning of a sentence S is taken to be included in every sentence S, then there exist no observation-sentences.)] You therefore reckon with the possibility of *observational* errors. But the premise that both *p* and *q* are asserted is not an observation-statement, "p is asserted" does not (or ought not to) mean the same as "p is not enclosed in quotes". However we explicate the meaning of "p is asserted", the explication ought to be restricted so as to make it true that the absence of quotation-marks (or other observables) does not guarantee that a sentence is used (i. e. asserted) — and also to make it true that the presence of quotation-marks does not guarantee that the sentence is mentioned.

LOG: You have a point there. There *is* a difference between saying about a particular specimen of the well-formed formula "p · ~p" that it is a contradiction, and saying about a particular specimen of the sentence "Norway is a democracy" in conjunction with a particular specimen of the sentence "Norway is not a democracy" that it is a contradiction.

I guess it is (at least in part) a matter of choice whether one says that the term "contradiction" does not mean the same in the two cases, or that the two sentences of the form "p is a contradiction" mean the same, but that the second one is harder to verify.

SEM: You now talk like a professional semanticist. Yet, since it is I who play the semanticist's part in this dialogue, let me play out his arguments — if not for your sake, then for the sake of our invisible spectators.

Not only does a logical analysis like our example include premises of the type "p is asserted". It also includes two other types of empirical — and hypothetical — premises. The one is about those parts of the text that are symbolized by *sentential connectives*, the other about those parts that are symbolized by *sentential variables*.

4. A logical analyst cannot escape premises about the relation between, say, a given instance of "or" and the usual semantical interpretation of the wedge, i. e. he cannot escape hypotheses about the probable meaning of "or" in a given context if his analysis is to be *of the text.*

If an analyst says, "I do not assert that this particular "or" is *in fact* inclusive, or probably so, only that if it is, then it can be symbolized by the wedge" — *and then symbolizes it by the wedge*, then either he does assert that the "or" is in fact inclusive, or his assertion does not justify his move (if it is *the text* he symbolizes). It's like if a person says, "If p, then q", and then concludes, "q": either p is an implicit premise or q does not follow.

If a sentence of the type "If p, then q" is taken to assert material implication from p to q, then it is true if only q is true. But if the point is to present p as a *justification* of q, we require that p shall be relevant to the subject matter, and also *true*.
If the analyst does not care about the actual or probable use of "or" in the text, then there is a recipe which may bring him some emotional satisfaction (*something* ought to guide his choice):

If a friend has written the text one says, "If this "or" is inclusive, then it can be symbolized by the wedge" — and then one symbolizes it by the wedge (and thus eliminates one possibility for contradiction). If an enemy has written the text one says (about the same "or"), "If this "or" is exclusive, then it can be symbolized by the symbol for non-equivalence" — and then one applies that symbol (and thus creates one more possibility for contradiction).

If an analyst does not want to create the illusion of justifying his moves, he may just say nothing or anything, e. g.: "If this "or" is a material implication, then it can be symbolized by the horse-shoe" — and then symbolize it by the wedge (if, for some reason, he sticks to the convention of symbolizing the word composed of the fifteenth and the eighteenth letter of the English alphabet, in that order, by the wedge).

5. *LOG*: Our spectators ought to notice that *SEM* speaks about the logical analyst whose job it is to find equivalences within a calculus to given items used outside the calculus. In his job the question of the material adequacy of a given symbolization (e. g. of representing a given instance of "if p, then q" by "$p \supset q$") is fundamental. With the pure logician the case is different, since the formal value of, e. g., the connective called "material implication" is independent of how well it fits with some frequent use of "if —, then —", which is how the horse-shoe is *read* very often.

SEM: Your distinction is worth a lecture of its own. Let there be a blackboard here, and a piece of chalk, too.

Consider those situations where there is a relation implied between, on the one hand, some connective like the horse-shoe, the wedge, etc. ("$\supset$", "$\vee$", etc.), call it an *F-element*, and, on the other hand, some item of English like "if —, then —", "or", etc., call it an *N-element*. Within such situations there is a variety of possible relations which may hold between the two elements in the pairs which are composed of one F-element and one N-element, e. g. in the pairs "$\supset$" and "if —, then —", "$\vee$" and "or". (We do not discuss pairs like "$\supset$" and "or".)

Some of these situations are relevant to our discussion. Consider the following:

(1) The *meaning* called “material implication” (i. e. the meaning attached to the horse-shoe when the sentential calculus is given its principal interpretation) is given. The job is to select some item of English to convey that meaning. (Imagine, e. g., an intelligent philosopher unfamiliar with mathematical logic saying to himself: I need a symbol to combine two sentences such that if the first is true and the second false, then the combination is false, otherwise it is true.)

Two possible responses to situation (1) are:

(11) One adopts the expression “if — then —”.

(12) One adopts the expression “if —, then —”, and attaches a recipe which tells the reader to interpret it as material implication.

In case of (11) one relies heavily on (even if one does not explicitly assert) the hypothesis that “if —, then —” is normally (within the relevant sort of context) used and interpreted in the sense of material implication. — In case of (12) one probably relies on the hypothesis that, of existing alternatives, “if—, then —” is closest (in some sense) to material implication.

A logician is sometimes in situation (1) when he tries to formulate professional assertions in English. Notice that English terms enter even into his axioms and theorems.

(2) The expression “if —, then —” occurs in an English sentence. The job is to give it a materially adequate symbolization within the propositional calculus.

Possible response:

(21) One symbolizes it by the horse-shoe (interpreted as material implication).

As in case (11) one relies heavily on an empirical hypothesis about the relation between “if —, then —” and material implication. The weakest hypothesis which if true justifies (21) is: the present instance of “if —, then —” is used in the sense of material implication. But the verification of this particular hypothesis is not independent of the verification of some general hypothesis like the one indicated in the comment on (11) above.

(2) is the situation of our logical analyst.

(3) The horse-shoe occurs *as an interpreted symbol* within the propositional calculus, e.g., in the formula “p ⊃ p”. The job is to desymbolize the formula, or at least the horse-shoe.

Possible response:

(31) One desymbolizes “p ⊃ p” as “if p, then p”.

The hypothesis implied is like that of (11), though situation (3) is not quite like situation (1). In (3) it is a symbol and the meaning attached to it which is given, in (1) it is the meaning only (irrespective of the shape of its symbol, and irrespective even of the existence of any (single) symbol with that meaning).

(4) The horse-shoe occurs as a symbol in the formal system called “the sentential calculus” or “the propositional calculus”. The job is to give an oral presentation of the system.

(41) The horse-shoe is read “if —, then —” (e. g. “p ⊃ p” is read “if p, then p”).

Case (4) is quite different from the cases (1)-(3). In (1)-(3) some *synonymity-relation* is implied (i. e. asserted by implication) to hold between the F-element and the N-element. In (4) the N-element is used either as a *name* of the F-element (insofar as the oral presentation of the system consists in talking about it), or it is used simply as a *reading device* (insofar as the presentation consists in oral reading of symbols and formulae): the N-element (as an item of the spoken language) is the *associated reading* of the F-element. (Human beings do not seem to be happy with logistic symbols unless some pronunciation has been matched to each of them. Even dots and dashes are read: * ‘dot-dot-dot-horseshoe-dash’ ’.)

One’s *motives* for choosing the reading “if —, then —” rather than “or” or “horse-shoe” may differ:

(a) because one believes that “if —, then —” is usually, or correctly, used in the sense material implication;
(b) because it is usual to pick the reading from the vernacular, and one believes that the meaning of “if — then” (in some of its frequent uses) is closer to material implication than is the meaning of other vernacular terms (like, e. g., or, and);
(c) because it is usual to read it like that, and one does not want to complicate matters by introducing variant readings.

Whatever one’s motives for choosing a particular reading, the very reading does not imply the assertion of any synonymity relation.

When the vernacular N-elements are used as names of logical connectives their status is similar to that of *test-names* in *psychology*: they are short labels which can be read without special instruction (since they are from the vernacular), and which have some mnemotechnic value since, we assume, each of them has been matched to the connective most similar to it in meaning (i. e. when the connectives are given their usual interpretation).

This, of course, is not the whole story — either for the test psychologist or for the pure logician. For the latter, who is not responsible for any particular interpretation of a formal system, there is still the theoretical question of the nature of such an interpretation.

LOG: I can think of an analogy, which, even if it does not reflect the network of distinctions in your lecture, still takes care of a major point.

Just as a painter may be abstract or naturalistic in relation to a given model, a logician may be abstract or naturalistic in relation to a given text. Very few propositions about the visual properties of an abstract painting are true about the visual properties of the model. But the absence of isomorphy between painting and model is no imperfection of either. If a series of English sentences is used as a stimulus in writing down a series of well-formed formulas, it is no defect of the formulas if they are not logically isomorphic to the English sentences (i. e., when the formulas are given their principal interpretation and the English sentences are given their probable meanings). Nor is it a defect of the English sentences. But a naturalistic portrait is imperfect if it does not resemble its model, and a logical analysis of a given text is defective if each well-formed formula is not isomorphic to the sentence it symbolizes.

SEM: Well, then my point is: In order to verify the existence of logical isomorphy between text and symbolization (take this wording in the sense you gave it above), one has to verify a number of empirical propositions about the actual meaning of the sentences in the text. We have discussed the premises about sentential connectives. The same sort of argument applies to our use of sentential variables.

Our premiss is then: *Sentences which are symbolized by the same sentential variable are synonymous, and sentences which are symbolized by different sentential variables are heteronymous.*

We do not know *a priori* that two specimens of the same sentence are synonymous any more than we know *a priori* that two specimens of different sentences are synonymous. I. e., our concept of 'synonymity' ought to be such as to imply what I just said. In the terminology of empirical semantics a given *term-type* is not *per definitionem* synonymous to itself. To make it so would not exactly amount to a denial of ambiguity, but it would collide with (other) existent terminologies in which the same term may be said to be used in different senses. A given *term-token* is synonymous to itself as used or interpreted by a given individual P_i at time t. Once P or t is varied, the interpretation of the given term-token is open to variation.

Hence, in order to justify the verdict "contradiction", it is not enough to cite two sentences (e. g., from the same text), and to point out that if the one has the form *p*, then the other has the form *not p*, since it is identical to the first except for a "not", etc. The observable properties of a pair of sentences furnish sufficient evidence only if the verdict is about observable properties (which it is not), or if there is an approximate one-to-one correlation between meanings and observable properties (which there is not).

The same is true for the verdict "tautology" or "analytic". A sentence of the form "S is analytic" is empirical — and hypothetical — whether S be a particular specimen of the sentence "No bachelor is married" or a particular specimen of the sentence "No unmarried man is married". The *meaning* of the sentence "S is analytic" is as clear or as unclear in the one case as in the other. The notion of synonymity is equally involved in both. On an intuitive interpretation of "S is analytic" I find the *likelihood* of analyticity greater for a given specimen of "No bachelor is married" than for a given specimen of "No unmarried man is married". My verdict is based on the belief that *whenever a given sentence is conspicuously analytic, it is probably not analytic* — making the thesis itself an example (almost) of what it asserts. It is my guess that research in empirical semantics will verify it.